## CONDITION MONITORING OF A CYCLOID GEARBOX

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**Abstract**: A cycloid drive for gearboxes allows for high reduction ratio and zero or very low backlash. The cycloid gear design is based on compression, whereas most gear interactions are based on shear. Further, the contract of a cycloid gear is typically subject to rolling forces vs. sliding, which are seen in traditional gearboxes. These features of a cycloid gearbox allow for high shock load capacity, high torsional stiffness, and quiet operation. This paper details the modeling required for correct configuration to perform analysis on the cycloid gearbox and then is demonstrated on a 51:1 ratio, run to failure test. This paper documents the sensitivity of standard condition indicators for gear/bearing during the run to failure test.

**Keywords**: Cycloid; Smart Sensor; vibration diagnostics; Resonance; Spectral Estimation; Model Bases Dynamics;

**Introduction**: There are many non-standard, condition monitoring application which use relatively unusual gearbox design. While most reduction (example: epicyclical) gearboxes have well-understood dynamics, other, such as a Variator (continuously variable transmission) or cycloidal gearbox, have not been reported on. This paper covers the dynamics, configuration and some test observation of work done on a cycloidal gearbox.

The interest in cycloidal gearbox is derived in that they are used in many applications were a low-cost drive motors is needed. For example, many conveyer belt systems (sorting, moving bulk media, slew drives) use cycloid drives. When the drive itself is a relatively low-cost asset, the process, which they support, can have significant economic impacts if they fail. For example, one of the larger courier delivery services has a distribution center with 3000 cycloid drives to move packages. The loss of a drive unit halts sorting, which could impact revenues of up to \$200,000 per day.

The cycloid drive for gearboxes allow for high reduction ratio and zero or very low backlash. The cycloid gear design is based on compression vs. shear forces, where the contact is typically rolling vs. sliding. These features allow for high shock load capacity, high torsional stiffness, and quiet operation. Single stage ratios more than 200:1 are possible.

The gearbox chosen for the test was an integrated induction motor and gearbox. This gearbox is rated for 0.75 kW, approximately 1 Hp drive. For 60 Hz operations, using a four-pole motor, the drive unit has a 100% input shaft rate of approximately 1795 rpm. The gearbox a 51:1 reduction ratio. Figure 1 shows the typical arrangement of a cycloid gearbox.



Figure 1. Typical Cycloid Gears (*copied from* <u>http://mechanicaldesign.asmedigitalcollection.asme.org/data/journals/jmdedb/27951/085</u> 001\_1\_1.jpeg)

The main components of the gearbox are: 1) inputs shaft, 2) support bearing, 3) eccentric bearings, 4) the cycloid gears, 5) the pin teeth-case, 6) the pins, 7) output rollers, 8) output shaft, 9) support bearing.

The ratio for the gearbox is given as:

ratio = 
$$\frac{(n_{\text{teeth}}-1) \times n_{\text{pins}}}{n_{\text{teeth}}-n_{\text{pins}}}$$
 (eq 1)  
(4) with 26 teeth and 51 pins

The test gearbox has dual disc (4) with 26 teeth and 51 pins.

**Equations of Motion and Configuration:** Configuration is driven by the equations of motions for the monitoring components. This consists of describing synchronous motion analysis of the shafts and gears, and the asynchronous motion of the bearings.

**Bearing Motion**: The simple input/output gearbox design uses three bearings on the input shaft: bearing D (the eccentric bearing) and input shaft bearing C. The output shaft was supported by two bearings: bearing B and bearing A.

The bearing rate fault frequencies are determined by the shaft rate, and:

- the number of rolling elements (*b*),
- the roller element diameter (*d*),
- the bearing pitch diameters (e), and
- the bearing contact angle ( $\alpha$ ).

The fault features are related to damage accumulated on the bearing itself.

There are typically six fault features calculated for the bearing associated with bearing elements: cage, ball, inner race, outer race. For mechanical looseness, the bearing may also generate signatures associated with whip/whorl (in the base spectrum) or a 1/revolution impact (tick) in the heterodyne analysis. The bearing feature rates are calculated as::

$$cage = 0.5(1 - d/e * \cos(\alpha))$$
 (eq 2)

$$ball = e/d (1 - (d/e)^2 * \cos(\alpha)^2)$$
 (eq 3)

inner race = 
$$b/2(1 + (d/e) * \cos(\alpha))$$
 (eq 4)

$$puter \ race = b/2 \left(1 - (d/e) * \cos(\alpha)\right) \tag{eq 5}$$

Because the outer rate of the eccentric bearing is in contact with the cycloid gear and the input shaft, the total rate seen by the bearing is the input shaft + output shaft. The eccentric bearing analysis was assigned to the input shaft. To capture the effect of the relative motion, the bearing rates were multiplied by 1 + 1/51 = 1.0196. This speed correction is needed in order to determine the correct bearing rate fault features.

**Shaft/Gear Motion**: Shaft and gear analyses are based on the time synchronous average, which requires an accurate rate from the tachometer. The tachometer is used to resample the vibration data and correct for any changes in shaft rate. Gear analysis, and more importantly gear mesh tones, is a function of the shaft rate and the number of teeth on the gear. A conventional input shaft with a 29.23 Hz rate with 26 teeth would have a gear mesh frequency of 29.23 x 26 = 759.96 Hz. However, the cycloid gear has a relative motion to the shaft, driven by the eccentric gear and the output shaft. The relative motion of the cycloid to the ring gear means that, for each revolution, there is one extra gear mesh. Hence, the actual gear mesh frequency would be 789.19 Hz. Figure 2 shows the corresponding TSA order of 27 (26 teeth + 1) and the accompanying raw spectrum (in Hz).

For this reason, gear analysis is based on 27 teeth, not 26 teeth.



Figure 2, Comparison of the Raw to TSA Spectrum for the input shaft.

The ring gear analysis would usually be associated with the number of ring gears teeth, but because there are two cycloid gears (of 26 teeth), the measured mesh is  $51 \times 2$  or 102 mesh. The TSA spectrum and raw spectrum then show frequencies at  $29.23/51 \times 102 = 58.46$  Hz. Due to the modulation of two disc, there are sidebands at  $102 \pm -51 = 51$  and 153, or 29.22 and 87.69 Hz, as seen in Figure 3.



Figure 3, Comparison of the Raw to TSA Spectrum for the output shaft.

**Window Selection for the Envelope Analysis**: The envelope analysis is based on demodulation of high-frequency resonance, which is based on impact. Poor selection of a window results in poor envelope/bearing analysis. Initially, with no faulted bearings to test, a "rap" test is conducted to excite a modal response. This is sampled, and the spectrum calculated, (see Figure 4 below).



Figure 4. Impact and resulting modal response.

Window selection is based on a sample rate of the sensor. The sample rate also affects the length of the TSA:

$$TSA \ length = 2^{ceil(log_2(Sample \ Rates/Shaft \ Rate))}$$
(eq 6)

Given the low output shaft rate of approximately 0.57 Hz, the measured acceleration will be low. For this reason, the acquisition length must be adequately long to capture perhaps 20 revolutions. Hence, a high sample rate taken over an extended period results and a very large data set, which takes more time to process and to download raw data (if needed).

For this reason, the sample rate of the output shaft was taken at 2930 sps, for 60 seconds. Note: at 0.57 Hz, this is only 34 revolutions. The TSA length is then 8192. For the input shaft, which is close to 30 Hz, only eight seconds of data was taken at 23438. This allows a Nyquist frequency of 1465 Hz for the output shaft, and 11719 for input shaft. From the model response in Figure 4, the window for output shaft analysis was taken at 300 to 1300 Hz, which covers the small resonant mode at 1000 Hz. For the input shaft, the window was taken from 9000 to 11000 Hz, covering the modal response at 10000 Hz (Figure 5).



Figure 5. Envelope Response from Impact

Gear Feature Analysis: The following CIs were used on the cycloidal gear:

Residual RMS, Residual Kurtosis, Residual Crest Factor, Energy Ratio, Energy Operator Kurtosis, Energy Operator Crest Factor, Figure of Merit 0, Side Band Lifting Factor, Side Band Analysis, Narrow Band Kurtosis, Narrow Band Crest Factor, Amplitude Modulation RMS, Amplitude Modulation Kurtosis, Frequency Modulation RMS, Frequency Modulation Kurtosis, Gear Mesh Energy [2].

**Health Indicator Paradigm:** In order to reduce maintainer workload, and improve fault detection, the concept of the health indicator was introduced in [2]. In general, the Health Indicator (HI) is a function of distributions of n CIs, where the HI is scaled such that:

- HI ranges from 0 to a positive value, where the probability of exceeding an HI of 0.35 is the PFA (probability of false alarm),
- A warning (yellow) alert is generated when the HI is greater than or equal to 0.75. Maintenance should be planned by estimating the RUL until the HI is 1.0.
- An alarm (red) alert is generated when the HI is greater than or equal to 1.0. Continued operations could cause collateral damage.

Note that this nomenclature does not define a probability of failure for the component, or that the component fails when the HI is 1.0. Instead, it suggests a change in operator behavior to a proactive maintenance policy: perform maintenance before the generations of collateral or cascading faults. For example, by performing maintenance on a bearing prior the bearing shedding extensive material, costly gearbox replacement can be avoided.

Hence, from a maintainer perspective, this is a stoplight-based threshold setting/alerting system: when a component is yellow, plan maintenance, and when the component turns red, do maintenance.

**Controlling for the Correlation Between CIs:** All CIs have a probability distribution (PDF). Any operation on the CI to form a health index (HI) is then a function of distributions. The HI function is defined as The norm of n CIs (energy):

$$HI = \frac{0.35}{crit}\sqrt{\mathbf{Y}^T\mathbf{Y}} \qquad (eq 7)$$

where Y is the whitened, normalized array of CIs, and *crit*, is the critical value.

The function is valid if and only if the distribution (e.g., CIs) are independent and identical (e.g., IID). As an example, for Gaussian distribution, subtracting the mean and dividing by the standard deviation will give identical Z distributions. The issue of ensuring independence is much more difficult. In general, the correlation between CIs is non-zero. For instance, Table 1 shows the correlation coefficients for 6 CIs used for gear fault analysis: most are statically significant.

	ρij	CI 1	CI 2	CI 3	CI 4	CI 5	CI 6
CI 1		1	0.84	0.79	0.66	-0.47	0.74
CI 2			1	0.46	0.27	-0.59	0.36
CI 3				1	0.96	-0.03	0.97
CI 4					1	0.11	0.98
CI 5						1	0.05
CI 6							1

Table 1: Correlation Coefficients for the Six CIs Used in the Study

This correlation between CIs implies that for a given function of distributions to have a threshold that operationally meets the design PFA, the CIs must be whitened (e.g., decorrelated). A whitening solution can be found using Cholesky decomposition. The Cholesky decomposition of Hermitian, positive definite matrix results in  $A = LL^*$ , where L is a lower triangular, and  $L^*$  is its conjugate transpose. By definition, the inverse covariance is positive definite Hermitian. It then follows that if:

$$LL^* = \Sigma^{-1}$$
, then  $Y = L \times CI^T$  (eq 8)

The vector CI is the correlated CIs used for the HI calculation, and Y is 1 to *n* independent CI with unit variance (one CI representing the trivial case). The Cholesky decomposition, in effect, creates the square root of the inverse covariance. This, in turn, is analogous to

dividing the CI by its standard deviation (the trivial case of one CI). In turn, it can be shown that  $\mathbf{Y} = \mathbf{L} \mathbf{x} \mathbf{CI}^{T}$  creates the necessary independent and identical distributions required to calculate the critical values for a function of distributions.

**Finding the Critical Value:** The critical value is taken from the inverse cumulative distribution function for the HI. The CIs used have Rayleigh like PDFs (e.g., heavily tailed). For magnitude based CIs, it can be shown that for the nominal case, the CI PDF is Rayleigh. For Gear CIs, and Bearing CIs (where magnitudes which are biased by RMS), a transform is used by make the CI more Rayleigh like. The transform "left shifts" the CI. For example, a shift such that the .05 CDF (cumulative distribution function) is assigned to 0.0.

Consequently, the HI function is based using the Rayleigh distribution. The PDF for the Rayleigh distribution uses a single parameter,  $\beta$ , defining the mean  $\mu = \beta^* (\pi/2)^{0.5}$ , and variance  $\sigma^2 = (2 - \pi/2) * \beta^2$ . The PDF of the Rayleigh is:  $x/\beta^2 \exp(x/2\beta^2)$ . Note that when applying these equations to the whitening process, the value for  $\beta$  for each CI will then be:  $\sigma^2 = 1$ , such that:  $\beta = \sigma^2 / (2 - \pi/2)^{0.5} = 1.5264$ .

For the HI equation in (7), the normalized energy of the CIs, it can be shown that the function defines a Nakagami PDF [2]. The statistics for the Nakagami are  $\eta = n$ , and  $\omega = 1/(2-\pi/2)*2*n$ , where *n* is the number IID CIs used in the HI calculation.

**Test Stand Results**: Using a nominal gearbox, run at approximately 50% load for 45 hours. Acquisitions were taken every 5 minutes, Figure 6.



Figure 6. Nominal Gearbox

**Initiation of Run to Failure**: After the initial test run to set threshold, the gearbox was run at 150% for one hour, then at 100% until failure. Note the resulting substantial step change. The step-change is seen on most components. It is likely that the eccentric bearing was damaged during the overload (Figure 7, 8).



Figure 7, Input Shaft driven by 1/Rev. Step change occurs during overload.

While the HI for both bearing C and D increase drastically (input shaft), it seems this is more a function of the generalized wear in the gearbox. This wear caused a low-frequency resonance within the gearbox, confounding the gear analysis for bearings A and B (Figure 9).



Figure 8, Bearing A (Output Shaft Bearing) health – likely not due to damage but measuring a noise floor increase due to damage in the high-speed section of the gearbox.

**General Comments on the Run to Failure Test:** The large number of failed components, as indicated by most of the components being in Alarm, suggests a high level of generalized wear. The high-speed input shaft shows an increase in SO1 (imbalance). It is suspected that there was damage to bearing C or D cage (output shaft bearings), allowing the shaft to no longer be centered. Again, this may have occurred during the overload (Figure 6).

The evidence to suggest that neither bearing A, nor B (output shaft) were damaged is that all four bearing components (Cage, Ball, Inner Race, Outer Race) are correlated – no

individual component indicating damage. This supports the hypothesis that the noise floor for the envelope analysis was increased due to damage in some other part of the gearbox.

The Eccentric bearing showed bearing wear due to mechanical looseness (as evident by the 1/Rev "Tick" at 29.75 Hz and a Ball failure, approximately 24 hours before failure (Figure 9)



Figure 9, Eccentric bearing Envelope analysis, one day before failure. Note 1/Rev (29 Hz) tone indicating mechanical looseness.

The unfiltered bearing health eventual spiked at a level of 100, one hour before failure. It is not unusual to see a dip in bearing fault feature energy with severe faults. This is due to the ball elements no longer rolling, but sliding – hence the slip is increased, and the frequency of the bearing fault feature drops, exceeding the search window of the bearing energy algorithm (Figure 10).



Figure 10. Eccentric bearing health spikes at 100.

The cycloid gear shows in alarm level gear mesh, from 50 to 20 hours before failure. Gear mesh energy is not usually a consistent indicator (Figure 11).



Figure 11. Cycloid Gear Health.

Note that from 20 hours before failure, Residual RMS, Energy Ratio, and FM0 is sensitive to the impending fault. From this, it was learned that the best five indicators for the cycloid gear health: Residual RMS, Energy Ratio, FM0, AM Kurtosis, and Gear Mesh. This suggests that during the last 10 to 20 hours of the run, the cycloid gear experienced a second failure mode that was detected by the more traditional gear faults (Figure 12).



Figure 12, Cycloid gear condition indicators

The ring gear showed damage as well. Because the gear health is based on the filtered HI, it does not reflect the step change HI of 2 in the raw HI data just prior to failure (Figure 13)



Figure 13: Ring Gear HI.

The gear CIs that were sensitive to this fault was found to be: Residual RMS, Energy Ratio, FM0, AM Kurtosis, and Gear Mesh. These condition indicators will be used in the future.

**Conclusion**: The cycloid gearbox has unique dynamics, which require using the correct ratios for the TSA and bearing rate calculation. The bearing analysis for the Cycloid gearbox is relatively standard. The eccentric bearing rate was multiplied by a correction factor to account for the rotating frame of the outer race. The eccentric bearing is the weak point of the gearbox. During the run to failure test, the eccentric bearing roller elements were found to be faulted. Towards the end of the run to failure test, the high level of damage (resonance energy) associated with the eccentric gear raised the noise floor of the envelope analysis, which contributed to high HI level of all the bearings in the gearbox.

The cycloid gear itself posed a challenge in that the apparent gear mesh frequency is based on the gear eccentric behavior and not on the gear mesh frequency alone. The observed gear mesh frequency is the gear tooth + 1 vs. gear tooth, multiplied by the shaft rate.

Because the gears are under compression vs. shear, care must be used in the CI selection used for the health indicator, as many of the traditional analyses were not sensitive. It was found that residual RMS, amplitude modulation RMS and gear mesh energy for gear tooth + 1 the shaft order worked well.

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[1] Eric Bechhoefer is the president of GPMS, Inc., a company focused on the development of low-cost condition monitoring systems. Dr. Bechhoefer is the author of over 100+ juried papers on condition monitoring and prognostics health management and holds 23 patents and five patent applications in the field of CBM.