RAPID IMPACT™ TESTING OF ANY SIZE STRUCTURE

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Abstract: One of the limitations of conventional modal testing using a roving impact hammer is that the ***reference sensor*** (usually an accelerometer) ***must remain fixed*** throughout the test. Since the accelerometer must typically be connected by a wire to the data acquisition system. a ***very long wire may be required*** when testing a large structure. Furthermore, ***better quality signals*** are possible if each ***impact force is applied closer to the response accelerometer***. Because it does not require ***a fixed reference sensor*** throughout the test, a **Rapid Impact™** test is faster and easier to perform on ***any size structure***.

Key words: Experimental Modal Analysis (EMA); Frequency Response Function (FRF); Impulse Response Function (IRF); Curve Fitting; Modal Residues; UMM Mode Shape; Multi-Input Multi-Output (MIMO) Modeling & Simulation; Modal Participation.

1. Introduction

In a conventional **Roving Impact** test, the ***accelerometer must remain fixed*** while the structure is impacted at different DOFs (points & directions). In a conventional **Roving Response Impact** test, the structure ***must be impacted at the same DOF*** while one or more accelerometers are moved to different points.

In **Rapid Impact™** testing, both the impact DOF and the accelerometer location can be changed between acquisitions. Based on their DOFs, a ***chain of acquisitions*** is made during a Rapid Impact™ test. A ***chain of FRFs*** is then calculated from the acquired data.

An FRF chain is not a conventional set of single-reference FRFs, yet it can be curve fit using single-reference curve fitting methods. However, the modal residues resulting from curve fitting the FRFs do not form a mode shape of residues. Instead, the residues must be further processed to obtain mode shapes. The residue post-processing is based on the ***relationship between modal residues and mode shapes***. This relationship is illustrated in Figure 13.

2. Roving Impact Test

A conventional **Roving Impact Test** is depicted in Figure 1. In this test, the accelerometer must remain fixed throughout the test, and the structure is impacted at a different DOF with each acquisition of data. Each set of impact & response data must be ***simultaneously acquired***. Each measurement set of ***simultaneously acquired data*** is used to calculate a **Frequency Response Function (FRF)**.

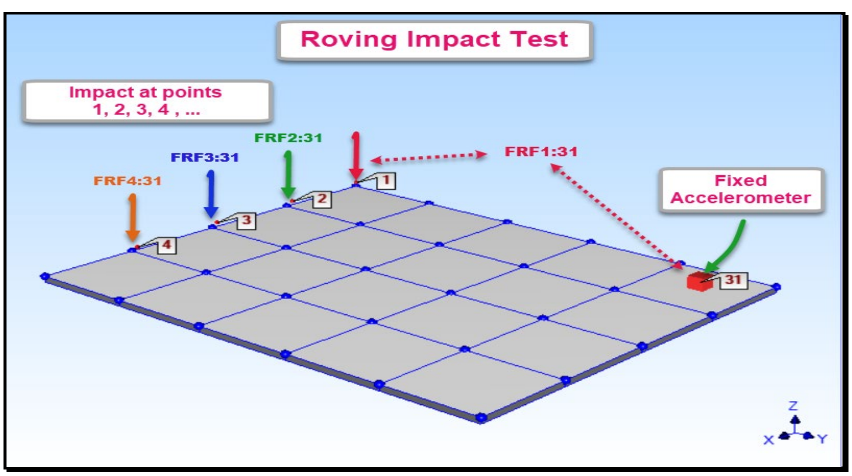

Figure 1. Roving Impact Test

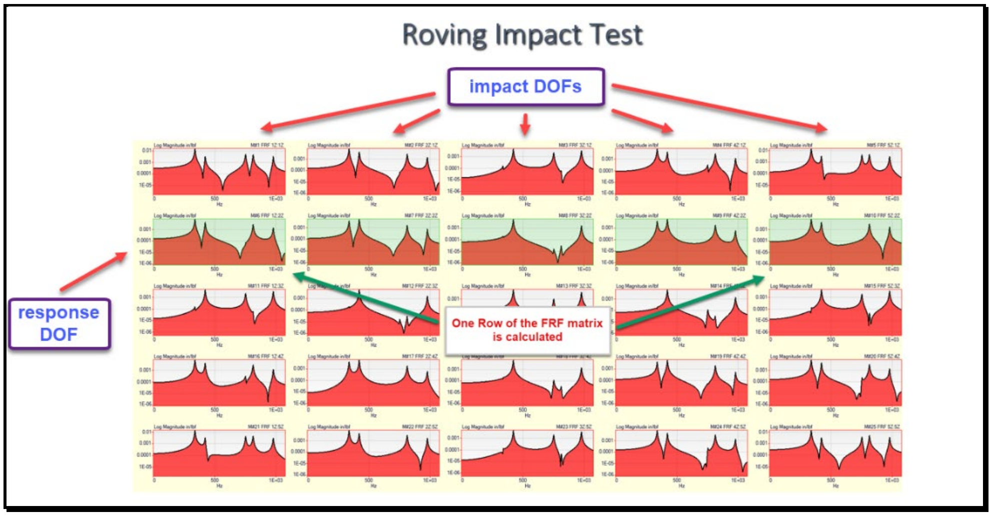

Figure 2. FRFs from a Roving Impact Test

The series of FRFs calculated from each measurement set of acquired data fill in a ***row of the matrix of possible FRFs***, as shown in Figure 2.

3. Roving Response Impact Test

The dynamics of the grating were originally captured with a ***conventional multi-reference roving accelerometer test***. Several photos of this test are shown in Figure 4. During this test, the grating was impacted in ***three directions at one corner*** **(DOFs 1X, 1Y, -1Z)**, and ***tri-axial accelerometers*** were attached to points on the grating where mode shape data was desired.

During this test, calibrated FRFs were calculated by entering the sensor sensitivities into the acquisition system so that each signal was converted from a voltage to its correct engineering units. Log magnitudes of several FRFs derived from this ***multiple-reference impact test*** are shown in Figure 3.

The properties of the multi-reference FRFs are listed in the **M#s spreadsheet** to the right of the log magnitude display. The FRFs have engineering units of (**g/lbf**). The log magnitudes show the resonance peaks of ***five modes of vibration.*** The mode shapes of these five modes were used to model the dynamics of the grating during a ***simulated*** Rapid Impact™ test of the grating. The results of this ***simulated*** Rapid Impact™ test are presented in the remainder of this paper.

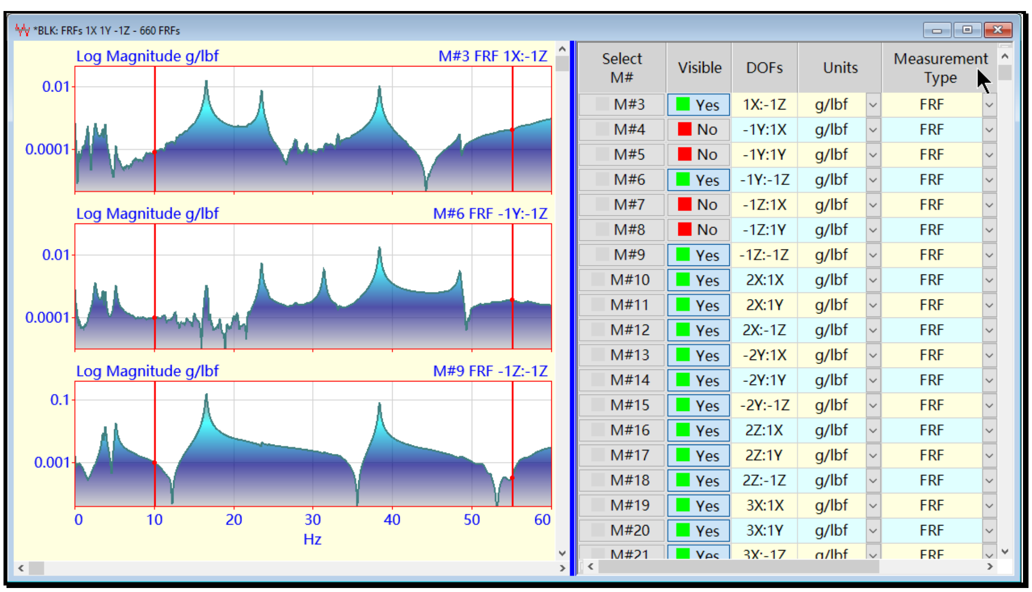

Figure 3. Multi-Reference Experimental FRFs of the Grating

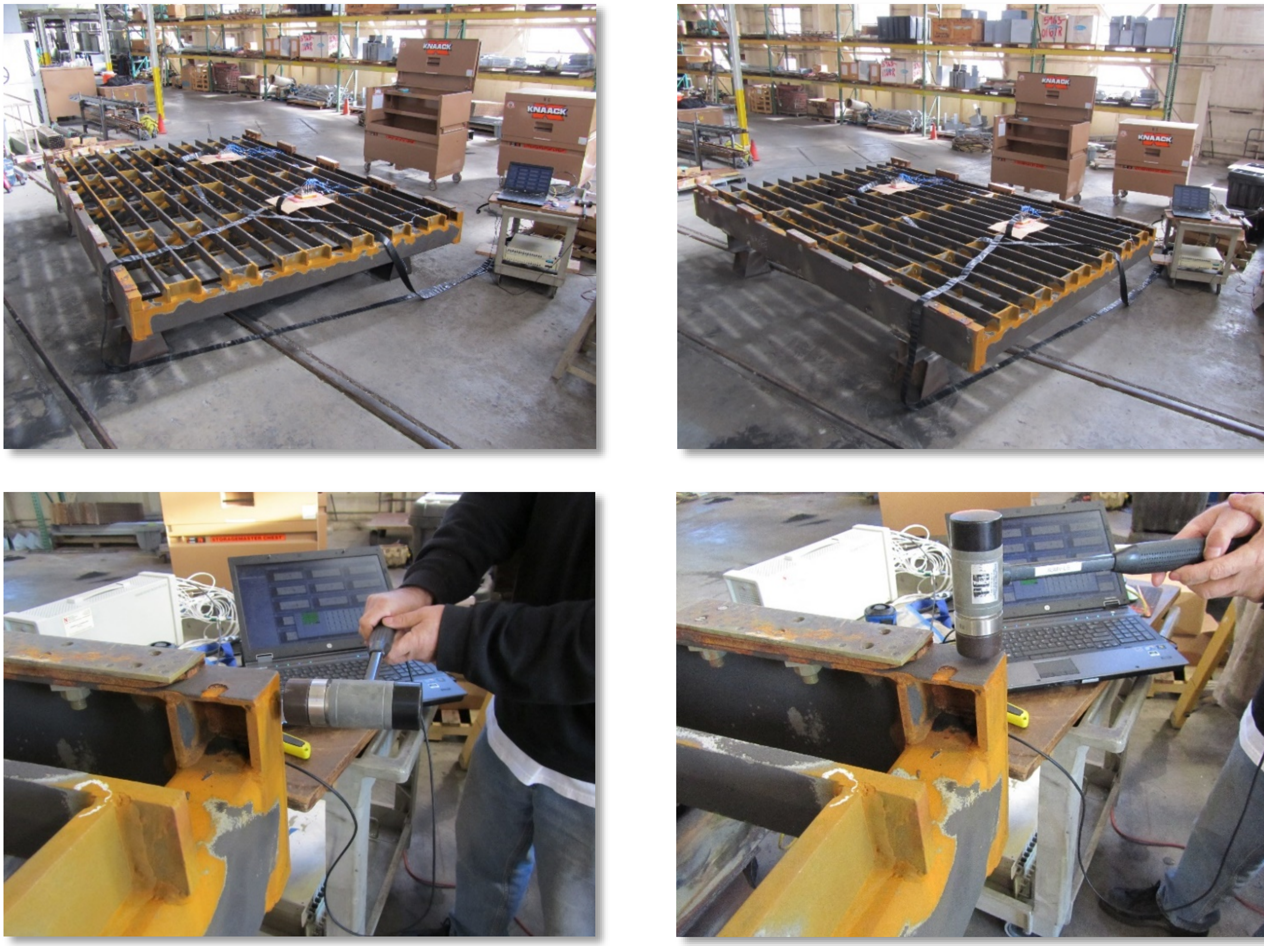

Figure 4. Multi-Reference Roving Accelerometer Test of the Grating

4. Mode Shapes of The Grating

The multi-reference FRFs acquired from the grating were curve fit using ***multi-reference curve fitting***. Because the FRFs were calibrated, the dynamic properties (mass, stiffness & damping) of the structure were preserved in the FRFs. The multi-reference modal residues obtained by curve fitting the FRFs also preserves the dynamic properties. Finally, the modal residues were used to create a **Modal Model (a set a of UMM mode shapes)** for the grating.

The ***truncated dynamic model*** of the grating consists of five **UMM mode shapes**. **UMM** stands for **Unit Modal Mass**, a special scaling of the mode shapes that also ***preserves the dynamic properties*** of the structure. This **Modal Model** was used to represent the dynamics of the grating during a ***simulated*** Rapid Impact™ test of the grating.

5. Modal Participation

One of the mode shapes of the grating is displayed in Figure 5. Also, the modal participations of each mode shape in each of the three references (1X, 1Y, -1Z) are listed in Figure 5 for the five mode shapes.

Modal participation indicates the direction of ***the dominant motion of a mode shape***. Modal participations are calculated during multi-reference curve fitting of a set of multi-reference FRFs. Modal participations have values ***between 0 & +-1***. **Participation = +-1** indicates ***dominate motion*** of the mode shape in the indicated direction. **Participation = 0** means the mode shape has ***no motion*** in the indicated direction.

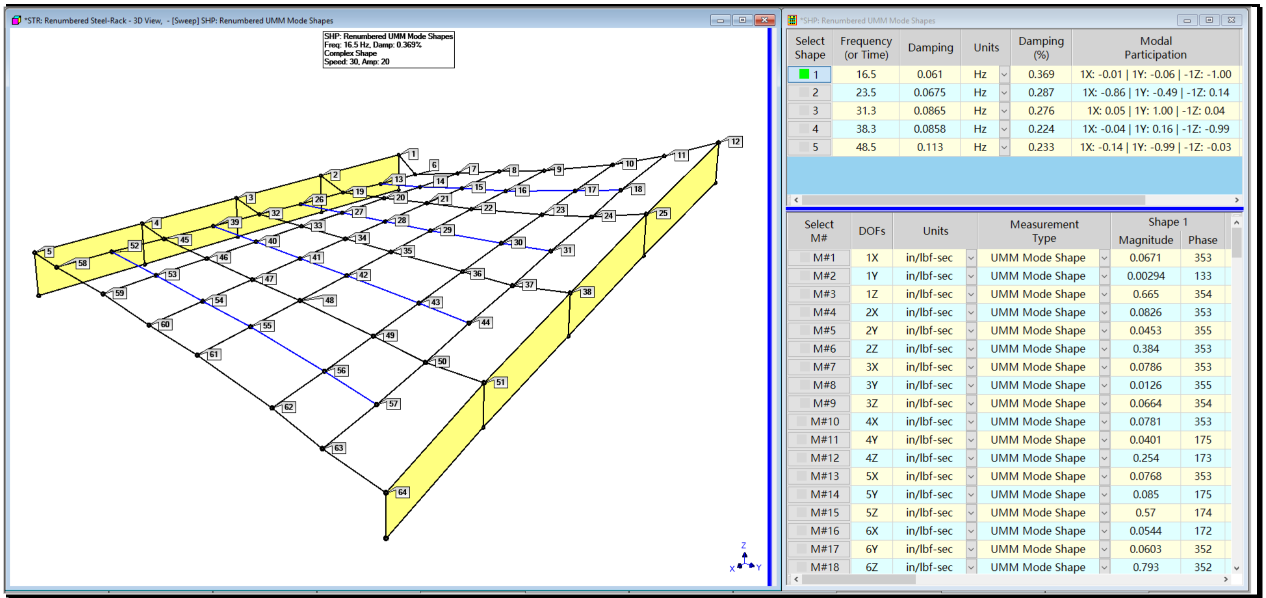

Figure 5. UMM Mode Shape of the Grating

The modal participations in Figure 5 indicate that ***modes 1 & 4 participate mostly in the Z-direction***. ***Mode 2*** participates ***mostly in the X-direction*** & ***Y-direction***, while ***modes 3 & 5 participate mostly in the Y-direction***.

Without knowing the dominant motion of each mode shape prior to a test, impact points & directions (DOFs) must be chosen to excite ***as many modes as possible***. In the original multi-reference impact test, the grating was excited in three directions at point #1, at DOFs (1X, 1Y, -1Z).

6. Simulated Rapid Impact Test™

The five UMM mode shapes of the grating were used to simulate a Rapid Impact test of the grating. Data acquisition from the grating ***was simulated*** as if it were acquired using an ***impact hammer,*** a ***tri-axial accelerometer*** and a ***4-channel acquisition system***.

Three random impact forces were applied in succession to the Y-direction & Z-direction at five points (numbered 1 to 5 and shown in the Figure below along one edge of the grating. The 3D response of the grating to the impacts was calculated for accelerometer positions in ***rows of points closest to each impact point***. The points in each row are numbered in succession, as shown in the Figure.

- When impacted at point 1, accelerations were calculated for points 6 to 18
- When impacted at point 2, accelerations were calculated for points 18 to 31
- When impacted at point 3, accelerations were calculated for points 31 to 44
- When impacted at point 4, accelerations were calculated for points 44 to 57
- When impacted at point 5, accelerations were calculated for points 57 to 64

Each response was calculated using MIMO Modeling & Simulation. This calculation is depicted in Figure 8. The simulation uses the **Modal Model of UMM mode shapes** to synthesize an FRF between each impact & response DOF. The first several impact impact measurements of the ***simulated*** Rapid Impact™ test are shown in Figure 6.

Four impact-response pairs are shown in Figure 9. Each impact-response pair shows ***three impacts*** and their resulting ***three impulse responses*** in the X, Y & Z-directions. As expected, a small impact caused a small response.

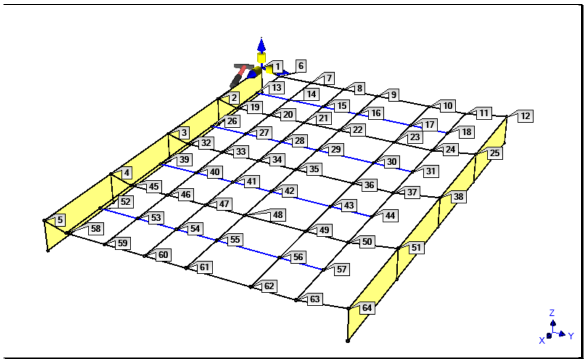

Figure 6a. First Acquisition

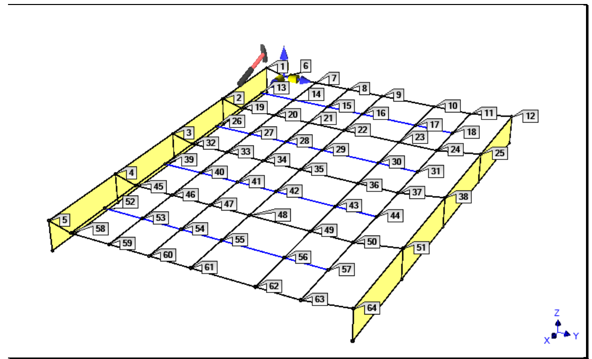

Figure 6d. Fourth Acquisition

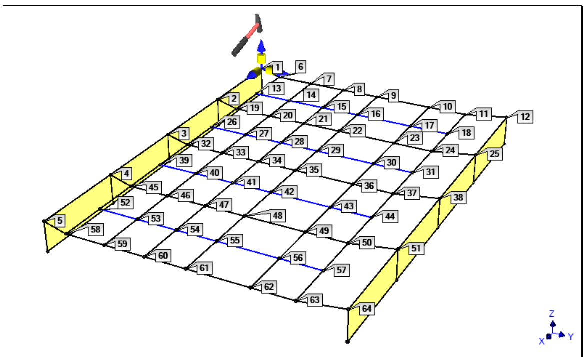

Figure 6b. Second Acquisition

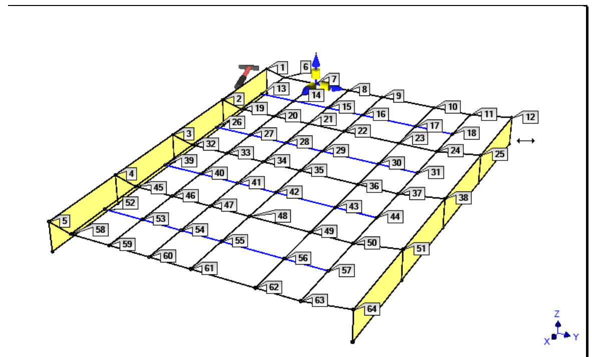

Figure 6e. Fifth Acquisition

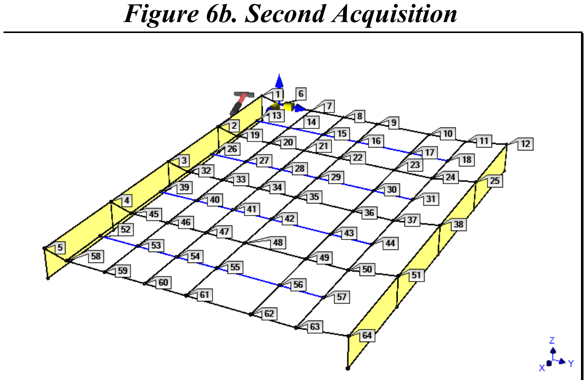

Figure 6c. Third Acquisition

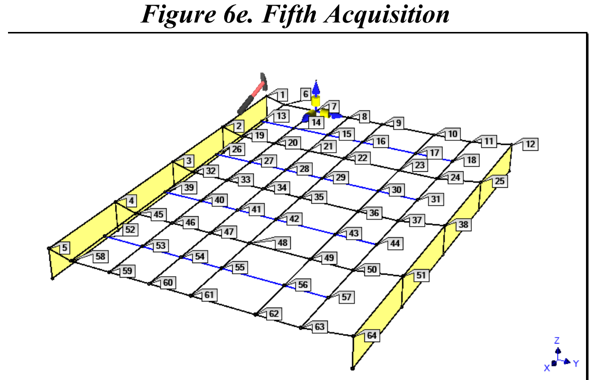

Figure 6f. Sixth Acquisition

During a Rapid Impact test, acquisitions can be made in ***any desired manner*** provided that a **chain of FRFs** can be calculated from the acquired data. An **FRF chain** is formed when each FRF has a ***Roving or Reference DOF that matches a DOF in another FRF***.

For this simulated Rapid Impact test, an FRF chain was calculated in three steps,

1. A sequence of three random impact forces was created for applying impacts in the Y-direction & Z-direction at each of five impact points along one edge of the grating. The random impacts simulate a real-world impact test. Several of these impact sequences are shown in Figure 7. Each impact force sequence contains **6144 samples** of time waveform data, enough to calculate three Fourier spectra with **1024 samples each**.

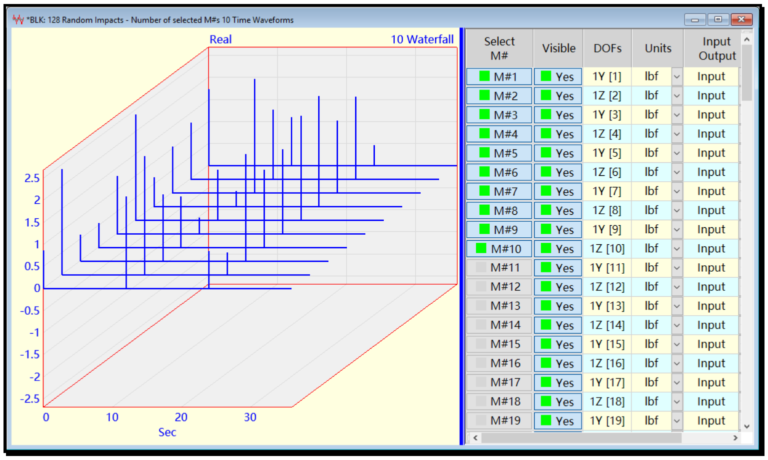

Figure 7. Impact Forces, Each Containing Three Random Impacts

2. Acceleration responses to each impact force were calculated in the X-direction, Y-direction & Z-direction. This calculation used the **Modal Model of UMM mode shapes** to represent the dynamics of the grating. FRFs were synthesized for each impact-response pair (input-output pair) using the UMM mode shapes. The MIMO calculation is depicted in Figure 8. Several impact-response pairs are shown in Figure 9. Each pair shows ***three impact forces*** and three corresponding ***impulse responses***. The responses are in the X, Y & Z-directions.
3. Data was ***"acquired"*** from the impact-response pairs shown in Figure 9. Each impact-response pair is given a ***unique*** **[Measurement Set number]** to define it as a ***simultaneous acquisition of 4 channels of data***.

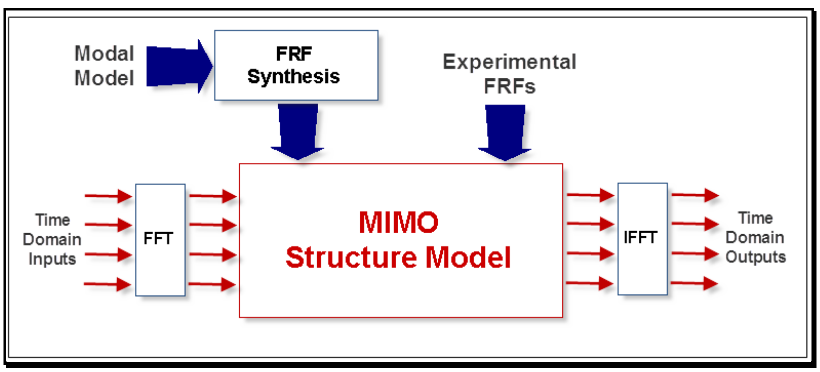

Figure 8. MIMO Modeling Diagram

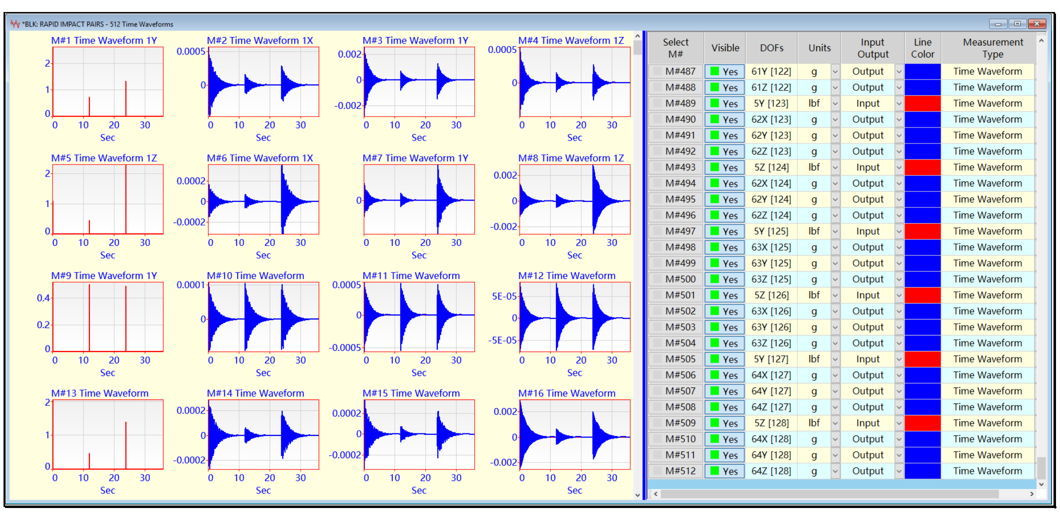

Figure 9. Rapid Test Impact-Response Pairs

7. Rapid Impact FRFs

To calculate FRFs, each **[Measurement Set]** of time waveforms was ***"acquiring"*** from the time waveforms shown in Figure 9. Three spectral estimates were averaged together using Stable (Linear) averaging, and **Coherence** was calculated together with each FRF. Each impact point & direction on the grating model is depicted with a hammer. Each accelerometer location is depicted with three arrows indicating the three responses of the tri-axial accelerometer. The acquisition process is depicted in Figure 10.

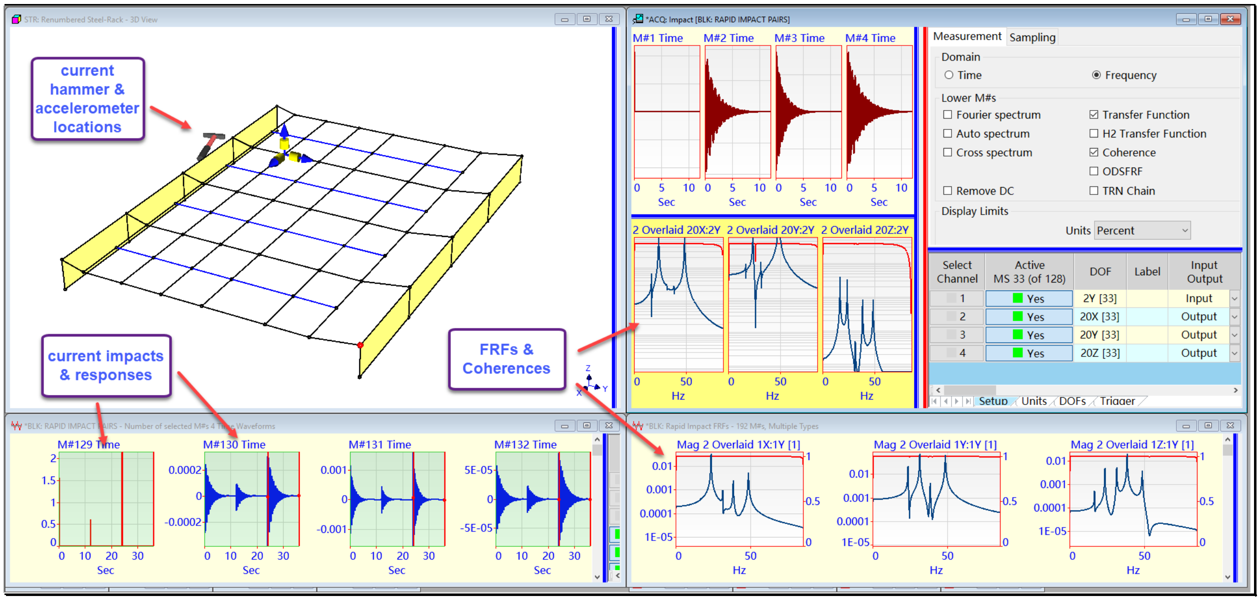

Figure 10. Rapid Impact FRFs & Coherence Calculation

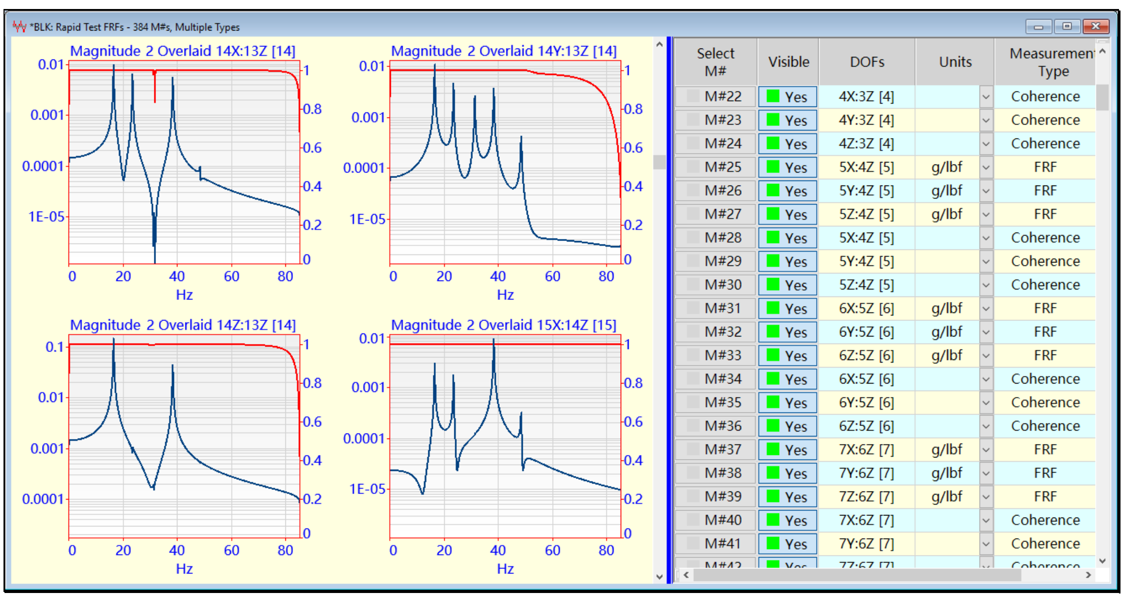

Figure 11. Rapid Impact FRFs & Coherence

408 FRF-Coherence pairs were calculated from the random impact-response pairs. Several FRFs, each overlaid with its corresponding Coherence, are shown in Figure 11. The FRFs represented measurements that would be calculated between impacts in the Y-direction & Z-direction (at 5 points along one edge of the grating), and 64 tri-axial accelerometer responses on the grating. The points are labeled in Figure 6.

Coherence ***values will drop below*** **"1"** at ***anti-resonances*** (low FRF values), ***near DC*** (zero frequency), and near the highest frequency in the FRFs. Figure 11 shows that very few Coherence values ***dropped below "1"*** indicating that the FRFs accurately measured the ***linear relationship*** between each impact force and one of its acceleration responses.

No special time domain windowing was required because the impact & response signals are both ***completely contained*** within each sampling window of time domain data (**2048 samples**). Hence, each time waveform is ***periodic in its sampling window***.

8. FRF Curve Fitting

The Rapid Impact FRFs were curve fit using the single-reference **Quick Fit** command. An example of a Quick Fit is shown in Figure 12.

Multi-reference curve fitting can only be used on FRFs that are ***multiple sets*** of ***single-reference FRFs***. To use multi-reference curve fitting, the FRFs for each reference DOF must contain the ***same Roving DOFs*** as all other FRFs with a different reference DOF. Stated differently, in order to use multi-reference curve fitting, the FRFs must fill ***multiple rows or columns*** of the FRF matrix of possible FRF measurements. Figure 2 shows an FRF matrix.

Following a curve fit, a **red Fit Function** is overlaid on each FRF. The **Fit Function** in Figure 12 closely matches the FRF, also indicated by **FRAC=1** on the ***upper right*** of the graph. **FRAC** is the frequency response version of **MAC** [4] that measures the ***co-linearity*** between an FRF and its **Fit Function**.

Each modal residue has the same DOFs as its corresponding FRF. Modal residues are displayed on the right side of Figure 12. **408 FRFs** were curve fit, so each mode has **408 modal residues**. Each modal residue has DOFs that match the DOFs of the FRF from which is was derived through curve fitting.

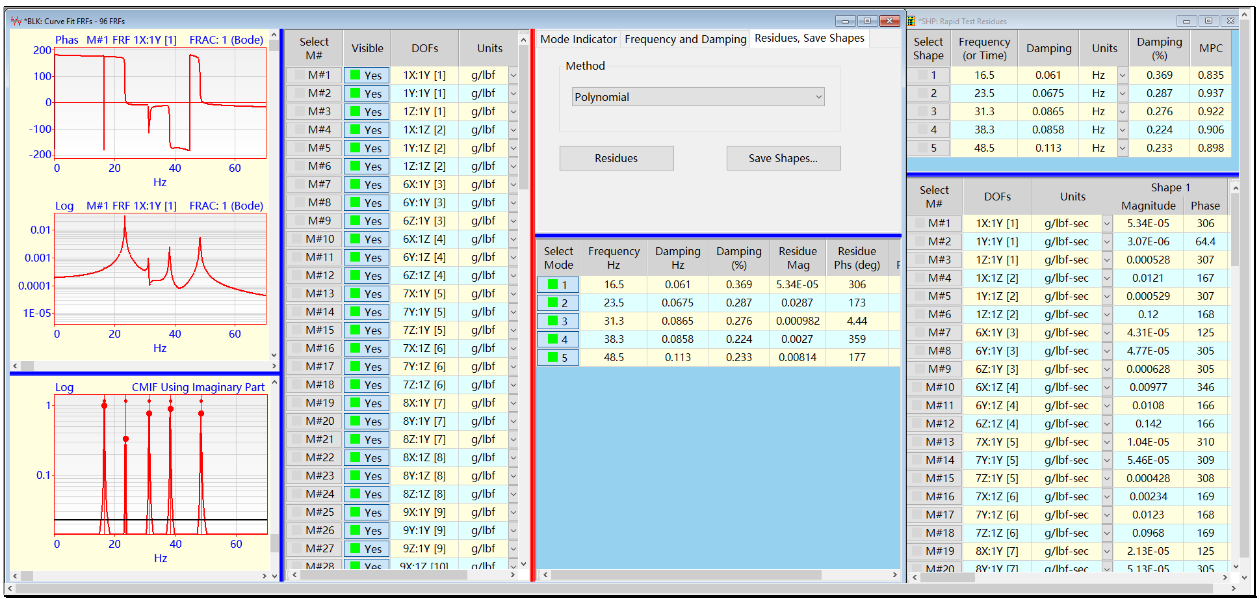

Figure 12. Quick Fit of Rapid Test FRFs

9. Modal Residues & Mode Shapes

Rapid Impact testing takes advantage of the mathematical relationship between modal residues and mode shapes. Figure 13 illustrates the relationship between modal residues and a mode shape.

Rapid Impact FRFs do not correspond to a ***row or column*** of FRFs in an FRF matrix, and therefore the modal residues obtained by curve fitting are ***not Residue Mode Shapes***. These modal residues must be further processed to obtain mode shapes, using the relationship "***Each modal residue is the product of two mode shape components***".

Figure 13 shows how modal residues obtained by curve fitting Rapid Impact FRFs are converted to a mode shape. In order to begin the conversion, a starting mode shape component is required.

The starting mode shape component is obtained either from a **driving point residue** or from **triangle point residues**. Using the starting mode shape component, all other mode shape components are computed using the relationship between residues and mode shape components.

Residues versus Mode Shape Components

$$\text{Residues} = \begin{Bmatrix} RES(1{:}2) \\ RES(1{:}3) \\ RES(2{:}3) \end{Bmatrix} \qquad \text{Mode Shape} = \begin{Bmatrix} MSC(1) \\ MSC(2) \\ MSC(3) \end{Bmatrix}$$

RES(1:1) = MSC(1) x MSC(1)

RES(1:2) = MSC(1) x MSC(2)

RES(1:3) = MSC(1) x MSC(3)

RES(2:3) = MSC(2) x MSC(3)

Single-Reference Modal Test

$$\text{Residue Mode Shape} = \begin{Bmatrix} RES(1{:}1) \\ RES(2{:}1) \\ RES(3{:}1) \end{Bmatrix} = \begin{Bmatrix} MSC(1) \text{ x } MSC(1) \\ MSC(2) \text{ x } MSC(1) \\ MSC(3) \text{ x } MSC(1) \end{Bmatrix}$$

Converting Rapid Test Residues to a Mode Shape

Starting with a Driving Point Residue

$$MSC(1) = \sqrt{RES(1{:}1)} = \sqrt{MSC(1) \text{ x } MSC(1)}$$

MSC(2) = RES(1:2) / MSC(1)

MSC(3) = RES(1:3) / MSC(1)

MSC(3) = RES(2:3) / MSC(2)

MSC(4) = RES(3:4) / MSC(3) = RES(4:3) / MSC(3)

Starting with Triangle Point Residues

$$MSC(1) = \sqrt{\frac{RES(1{:}2) \text{ x } RES(1{:}3)}{RES(2{:}3)}}$$

$$= \sqrt{\frac{MSC(1) \text{ x } MSC(2) \text{ x } MSC(1) \text{ x } MSC(3)}{MSC(2) \text{ x } MSC(3)}}$$

Figure 13. Modal Residues & Mode Shapes

10. Animated Mode Shape Comparison

As a ***"round trip"*** comparison, the Rapid Test mode shapes were compared in animation with the original UMM mode shapes. A comparison of a Rapid Test mode shape on the left and a UMM mode shape on the right is shown in Figure 14.

MAC & SDI Bars: Notice that **Modal Assurance Criterion (MAC)** and **Shape Difference Indicator (SDI)** bars are also displayed with each shape pair. These two correlation coefficients have the following properties,

- MAC is a measure of the ***co-linearity*** between two shapes [4]]
- SDI is a measure of the ***difference*** between two shapes [3]
- Both MAC & SDI have values between 0 &1
- A MAC or SDI value ***greater than 0.9*** indicates a ***strong correlation*** between two shapes
- A MAC or SDI value ***less than 0.9*** indicates a ***weak correlation***

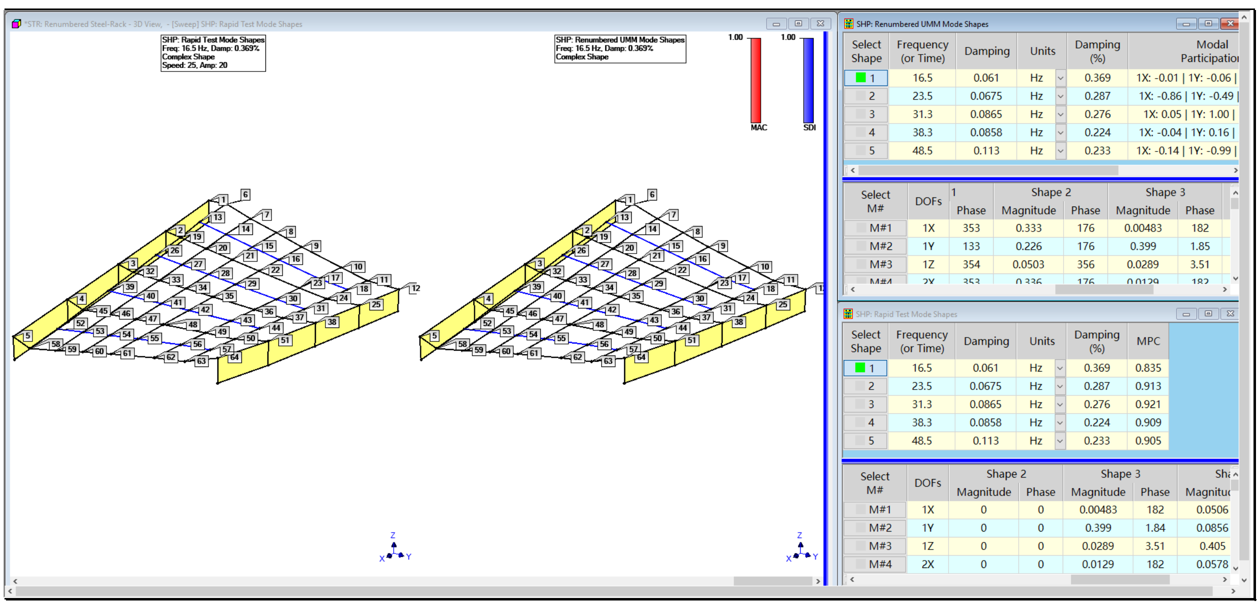

Figure 14. Rapid Test & UMM Mode Shapes

From the animated shape comparison (including the MAC & SDI bars), it was evident that the modal parameters (frequency, damping & mode shape) of all five Rapid Test modes ***matched the parameters of the original UMM modes very closely***.

11. Summary of The Rapid Impact™ Test

A Rapid Impact™ test ***was simulated*** on a grating from a water treatment plant. It was assumed that an **impact hammer**, **tri-axial accelerometer**, and **4-channel simultaneous data acquisition** were used for the test. The following steps were carried out in this simulation,

Step [1] - 136 random impact forces were created to simulate real-world impacting of the grating in the Y-direction & Z-direction at 5 points

Step [2] - Impact-response pairs were calculated using **MIMO Modeling & Simulation** and a **Modal Model of five UMM mode shapes** to represent the dynamics of the grating between pairs of impact & response DOFs

Step [3] – **Frequency Response Functions (FRFs)** and their corresponding **Coherences** were calculated from each impact-response pair. 408 FRF & Coherence pairs were calculated

Step [4] - The Rapid Impact™ FRFs were curve fit to obtain the modal frequency, damping, and **modal residues** for five modes of the grating

Step [5] - The Rapid Impact **modal residues** were converting into **mode shapes** of the grating

Step [6] - The Rapid Test **mode shapes** were compared in animation with the original **UMM mode shapes**

Rapid Impact™ testing offers several advantages over conventional single-reference or multi-reference impact testing,

1. In a conventional impact test, the ***reference sensor*** (either the impact DOF or the accelerometer location) ***must remain fixed throughout the test***. Since the reference sensor must be connected by a wire to the data acquisition system. a ***very long wire may be required*** when testing a large structure.
2. *B**etter quality signals** are possible if each impact force **is applied closer to the response accelerometer***
3. In a Rapid Impact™ test, either the ***impact hammer or the accelerometer can be moved*** to a different DOF between acquisitions of data. One sensor can remain fixed while the other one is moved between acquisitions, thus forming a ***chain of acquisitions*** based on their DOFs
4. A Rapid Impact™ test is faster and more convenient to use on ***any size structure***

During a Rapid Impact™ test, a chain of acquisitions is formed based on their DOFs. A chain of FRFs can then be calculated from the chain of acquired data.

Each FRF has two DOFs associated with it. An ***FRF chain*** is formed when the ***Roving or Reference DOF of each FRF has the same DOF as another FRF in the chain***.

After the Rapid Test FRFs are curve fit, the modal residues can be converted to mode shapes using the special relationship, ***"Each modal residue is the product of two mode shape components"***.

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